

Perfect Cubes

For hundreds of years Fermat's Last Theorem, which stated simply that for $n > 2$ there exist no integers $a, b, c > 1$ such that $a^n = b^n + c^n$, has remained elusively unproven. (A recent proof is believed to be correct, though it is still undergoing scrutiny.) It *is* possible, however, to find integers greater than 1 that satisfy the "perfect cube" equation $a^3 = b^3 + c^3 + d^3$ (e.g. a quick calculation will show that the equation $12^3 = 6^3 + 8^3 + 10^3$ is indeed true). This problem requires that you write a program to find all sets of numbers $\{a,b,c,d\}$ which satisfy this equation for $a \leq 100$.

The output should be listed as shown below, one perfect cube per line, in non-decreasing order of a (i.e. the lines should be sorted by their a values). The values of b, c , and d should also be listed in non-decreasing order on the line itself. There do exist several values of a which can be produced from multiple distinct sets of b, c , and d triples. In these cases, the triples with the smaller b values should be listed first.

Note that the programmer will need to be concerned with an efficient implementation. The official time limit for this problem is 2 minutes, and it is indeed possible to write a solution to this problem which executes in under 2 minutes on a 33 MHz 80386 machine. Due to the distributed nature of the contest in this region, judges have been instructed to make the official time limit at their site the greater of 2 minutes or twice the time taken by the judge's solution on the machine being used to judge this problem.

The first part of the output is shown here:

Cube = 6, Triple = (3,4,5)
Cube = 12, Triple = (6,8,10)
Cube = 18, Triple = (2,12,16)
Cube = 18, Triple = (9,12,15)
Cube = 19, Triple = (3,10,18)
Cube = 20, Triple = (7,14,17)
Cube = 24, Triple = (12,16,20)